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Franke, Reiner

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Artificial Long Memory Effects in Two Agent-Based Asset Pricing Models

by Reiner Franke



Christian-Albrechts-Universität Kiel

Department of Economics

Economics Working Paper

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Artificial Long Memory Effects in Two Agent-Based Asset Pricing Models

Reiner Franke^{a,*}

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^aUniversity of Kiel, Germany

Abstract

This note is concerned with two recent agent-based models of speculative dynamics from the literature, one by Gaunersdorfer and Hommes and the other by He and Li. At short as well as long lags, both of them display an autocorrelation structure in absolute and squared returns that comes remarkably close to that of real data at a daily frequency. The note argues that these long memory effects are to be ascribed to the stochastic specification of the price equation, which given the wide fluctuations in these models unduly fails to normalize the price shocks. Under an appropriate respecification, the long memory completely disappears.

JEL classification: C15; D84; G12.

Keywords: Volatility clustering; Autocorrelations of returns; Fundamentalists and trend-followers.

1. Introduction

Volatility clustering and long memory effects are among the most important ‘stylized facts’ in (daily) financial time series data. As evidenced by insignificant autocorrelations (ACs) of raw returns and a hyperbolic decline of the ACs of the absolute and squared

* Corresponding author.

Email address: `franke@iksf.uni-bremen.de` (*Reiner Franke*).

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returns, these features have spurred attempts at a theoretical explanation. Since traditional economic and finance theory based on the representative agent with rational expectations had been fraught with difficulty in this respect, there has been an increased interest in models incorporating heterogeneous types of agents and some form of ‘market psychology’ or ‘investor sentiment’.

Originating with Beja and Goldman (1980), a wide class of these models displays just two archetypical groups of speculative traders, namely, fundamentalists and trend-followers. The fact that fundamentalists tend to stabilize and trend-followers tend to destabilize the market offers broad scope for oscillatory price dynamics. If in addition the weights of the demand of the two groups on the market are suitably varying over time, there may be good prospects of generating the desired AC patterns as an endogenous phenomenon. In fact, there are two recent papers that claim to have achieved this goal. One is Gaunersdorfer and Hommes (2007; GH henceforth), where the variations of the weights are governed by a process of evolutionary fitness for the market fractions of the two groups and the fitness is measured by the accumulated realized profits. The other paper is He and Li (2007; HL henceforth). They fix the market fractions of the traders, and variations of their impact on the market are brought about by risk adjustments in the demand of the trend-followers, which in turn derive from a geometric learning process.

While one can find some other low-dimensional models in the literature that also purport to match the stylized facts to some degree, it may, in short, be said that they have a poorer and less satisfactory theoretical structure.² In comparison, the theoretical design of GH and HL provides a fruitful and convincing compromise of rich, substantial and yet parsimonious modelling. Within a small-scale agent-based framework, the two papers appear to be the best theoretical explanation of the empirical AC patterns that are currently available. The present note, however, sets out to spoil this positive evaluation. It will argue that in both models the long memory effects are to be attributed to a particular specification of the exogenous stochastic perturbations in the price equation, which is arbitrary and even artificial given that it unduly fails to normalize this noise. Furthermore, once a respecification of the noise takes an obvious normalization into account, the long memory effects will completely disappear.

It is useful for the presentation to begin the discussion with the GH model, which is done in Section 2. Subsequently, Section 3 deals with the HL model, and Section 4 concludes.

² We are thinking of Westerhoff (2003), Manzan and Westerhoff (2005), or Alfarano et al. (2005).

2. The Gaunersdorfer–Hommes model

2.1. Formulation of the model

The Gaunersdorfer–Hommes (GH) model employs the standard framework with one risky asset, i.e. a large stock or market index, that pays a (stochastic) dividend y_t per share at the beginning of the market period t , and risk-free bonds that pay a fixed rate of return r . The demand $z_{h,t}$ for the risky asset by trader type h is supposed to be determined by the expected excess returns,

$$z_{h,t} = z_{h,t}(p_t) = \frac{E_{h,t}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a\sigma^2} \quad (1)$$

where p_t is the stock price (ex-dividend) in period t , $E_{h,t}$ are the conditional expectations of trader type h , a is a uniform risk aversion coefficient, and σ^2 is the conditional variance, which is here supposed to be uniform across all traders and constant over time.³ Dividends are correctly expected to be given by $\bar{y} > 0$, so the agents know the true fundamental value

$$p^* = \sum_{\tau=1}^{\infty} \frac{\bar{y}}{(1+r)^\tau} = \frac{\bar{y}}{r} \quad (2)$$

The market is populated by two types of speculative traders: *fundamentalists* (type 1) and *trend-followers* (type 2). The price expectations for the next period $t+1$ are:

$$\begin{aligned} E_{1,t}(p_{t+1}) &= p^* + v(p_{t-1} - p^*), & 0 \leq v \leq 1 \\ E_{2,t}(p_{t+1}) &= p_{t-1} + g(p_{t-1} - p_{t-2}), & g \geq 0 \end{aligned} \quad (3)$$

A Walrasian auctioneer takes care of market clearing. With respect to predetermined market fractions $n_{h,t}$ of the two types of traders ($n_{1,t} + n_{2,t} = 1$), he sets the period- t price p_t such that $\sum_{h=1}^2 n_{h,t} z_{h,t}(p_t) = z^s$, where the supply z^s of the asset is assumed to be fixed and, for convenience, equal to zero. This condition can be explicitly solved for p_t . Introducing at this place also i.i.d. additive price shocks (APS), which are normally distributed with standard deviation σ_ε , we get

$$\hat{p}_t := \frac{1}{1+r} \left[p_{t-1} + \bar{y} + \frac{n_{1,t}(1-v)(p^* - p_{t-1}) + n_{2,t}g(p_{t-1} - p_{t-2})}{a\sigma^2} \right] \quad (4)$$

$$p_t = \hat{p}_t + \sigma_\varepsilon \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \quad (\text{APS})$$

The core of our criticism of the GH model will be that this specification of a random influence on price formation is less innocent than it might look at first sight.

The changing composition of the agents is governed by an evolutionary process. The market fraction $n_{h,t}$ of trader type h in period t is based on the fitness $U_{h,t-1}$ of this

³ Usually (1) is said to be derived from a mean-variance optimization of expected wealth. In this case ‘demand’ $z_{h,t}$ is the agents’ desired holding of the asset; see Franke (2008) for a clarification of the concepts behind eq. (1).

strategy in the previous period, which derives from information up to $t-1$. The standard discrete choice probability for the trend-followers is then determined as follows,

$$\tilde{n}_{2,t} = \frac{\exp(\beta U_{2,t-1})}{\sum_{h=1}^2 \exp(\beta U_{h,t-1})} = \frac{1}{1 + \exp[\beta (U_{1,t-1} - U_{2,t-1})]} \quad (5)$$

where the parameter $\beta > 0$ is the well-known intensity of choice. It is a special feature of the GH model that the actual market fraction of the trend-followers is not directly given by (5). GH rather assume that the more the market price diverges from the fundamental value, more and more trend-followers start believing that a price correction is about to occur. This idea is captured by a dampening factor. That is, $\tilde{n}_{2,t}$ is multiplied by a positive coefficient (equal to or) less than unity, which decreases as p_{t-1} moves further away from p^* . Letting the strength of the dampening mechanism depend on a parameter $\alpha > 0$, the market fractions of the two types of agents are thus specified as,

$$n_{2,t} = \frac{\exp[-(p_{t-1} - p^*)^2/\alpha]}{1 + \exp[\beta (U_{1,t-1} - U_{2,t-1})]} \quad (6)$$

$$n_{1,t} = 1 - n_{2,t} \quad (7)$$

It remains to present the fitness function, which is akin to discounted profits. Precisely,

$$U_{h,t-1} = [p_{t-1} + y_{t-1} - (1+r)p_{t-2}] z_{h,t-2} + \eta U_{h,t-2} \quad (0 \leq \eta \leq 1) \quad (8)$$

The term in square brackets is the excess return per share of the risky asset over the risk-free asset, which trader type h has realized in period $t-1$. It is multiplied by the demand from the period before that, which yields the sum of his excess profits for period $t-1$. These, in turn, are added to the fitness from the previous period, where η is a memory parameter that measures how slowly past fitness is discounted.⁴

Equation (8) completes the description of the model. Directly, the actual market price p_t in (4) and (APS) depends on the two lagged prices p_{t-1} and p_{t-2} . However, the population shares $n_{h,t}$ entering there depend on the fitness $U_{h,t-1}$, which besides p_{t-1} , p_{t-2} depends on the forecasts $E_{h,t-2}$; see (5)–(8) and (1), the latter correspondingly dated backward. Then (3) shows that also p_{t-3} and p_{t-4} are used in (8). On the whole, with $U_{1,t-1}$, $U_{2,t-1}$ and four lagged prices in the market clearing condition (4), the deterministic skeleton is six-dimensional. Given initial values of these six variables, the stochastic model can, of course, be easily iterated forward.

2.2. Volatility clustering and long memory

In model (1)–(8), price changes are driven by a combination of additive random forces acting on the market price and an evolutionary mechanism governing the proportions in which the two trading strategies are adopted. With a suitable calibration, the asset

⁴ A more obvious formalization of the idea of discounting would be the weighted average $(1-\eta)[\dots] z_{h,t-2} + \eta U_{h,t-2}$. Presumably, eq. (8) simplifies the mathematical analysis of the model.

market turns out to be characterized by an irregular switching between phases of low volatility, where the price changes remain small, and phases of high volatility, where initially small price changes caused by the random perturbations are reinforced and may eventually become large due to the trend-following traders. Hence volatility clustering arises, brought about by heterogeneity and conditional evolutionary learning. In addition, the model is able to generate autocorrelation patterns of raw returns as well as absolute and squared returns that bear some similarity to those observed in daily data of a stock market index like S&P 500.⁵

$a\sigma^2$	\bar{y}	r	v	g	β	α	η	σ_ε/p^*
1	1	0.001	1.00	1.90	2.00	1800	0.99	0.01

Table 1: Numerical parameters of the GH model.

The numerical parameter values by which these results have been achieved are collected in Table 1. The first 15,000 periods of a sample run are plotted in Figure 1. From the brief summary it is clear that they have to be interpreted as days. The time series thus spans $15,000/250 = 60$ years. The plus sign added to ‘APS’ in the caption of the figure will be explained further below.

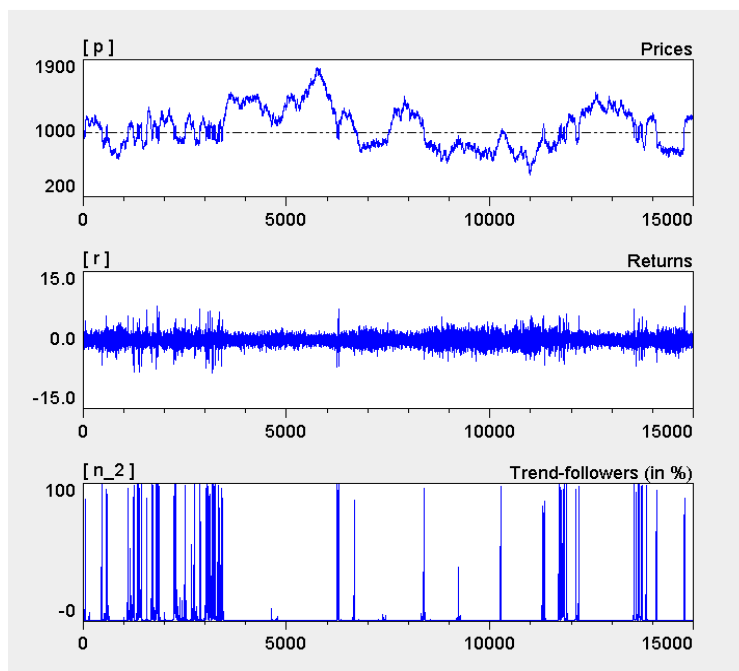


Figure 1: Simulation of the GH model with additive price shocks (APS+).

⁵ This summary paraphrases GH (2007, p. 266).

One does not need to compute precise statistical numbers in order to see that the return series r_t in Figure 1 exhibits a strong pattern of volatility clustering.⁶ As GH (2007, p. 283) point out, the key parameter on which this result depends is the coefficient v in the expectations of the fundamentalists. It must be close to or, as in the present case, equal to one, while the phenomenon is reported to be fairly robust with respect to variations in the other parameters. Conceptually, $v=1$ means that the fundamentalists in this model are rather believers in the efficient market hypothesis (EMH), since the naive forecast $E_{1,t+1} = p_{t-1}$ does not refer to any fundamental value at all and is consistent with an efficient market where prices follow a random walk. GH also notice that if all agents are EMH believers, $n_{1,t} = 1$ in \hat{p}_t in (4), the market price equation (APS) becomes

$$p_t = \frac{p_{t-1} + r p^*}{1 + r} + \sigma_\varepsilon \varepsilon_t = p_{t-1} + \frac{r}{1 + r} (p^* - p_{t-1}) + \sigma_\varepsilon \varepsilon_t \quad (9)$$

Given the small magnitude of the daily interest rate $r = 0.001$, the price dynamics is close to a random walk under these circumstances, which would only confirm the trading strategy of the agents.

On this basis the occurrence of volatility clustering can be explained as follows. When EMH believers dominate the market, prices are highly persistent and essentially driven by the exogenous shocks ε_t . A comparison of the first and third panel in Figure 1 shows that such a regime prevails as long as the price keeps a certain distance away from the fundamental value $p^* = \bar{y}/r = 1000$. It will become clear in a moment that quite independently of the differential fitness in (5), the near extinction of trend-followers in these phases is due to the dampening in eq. (6).

Accidentally, however, the price returns into a vicinity of p^* . The corresponding trending behaviour in the price lets the trend-followers earn higher profits. At the same time there is a weaker dampening in (6), so that the market fraction $n_{2,t}$ of the trend-followers rises. This, in turn, reinforces the current upward or downward motion in the price, which additionally favours the trend-following strategy. As a consequence, the destabilizing effects of this trading rule amplifies the price changes and these stages are characterized by excess volatility.

The succession of the tranquil and volatile phases can be studied in greater detail if we consider the 81-day subinterval between $t = 6,230$ and $t = 6,310$ for p_t , r_t and $n_{2,t}$. This is done in Figure 2. It is here evident that trend-followers are in the majority if—and only if—the price is close to p^* . The relatively gradual increase of $n_{2,t}$ as p_t approaches p^* , and the precipitous fall after p_t has crossed the $p = p^*$ line, is typical for the evolutionary dynamics and certainly an attractive feature of the model. The emergence of this asymmetry can be best understood in the deterministic framework, where because of the greater smoothness it is even more pronounced (see Figure 1 and its discussion in GH, 2007, pp. 277f).

⁶ Returns are defined as $r_t = 100 \cdot (p_t - p_{t-1})/p_{t-1}$.

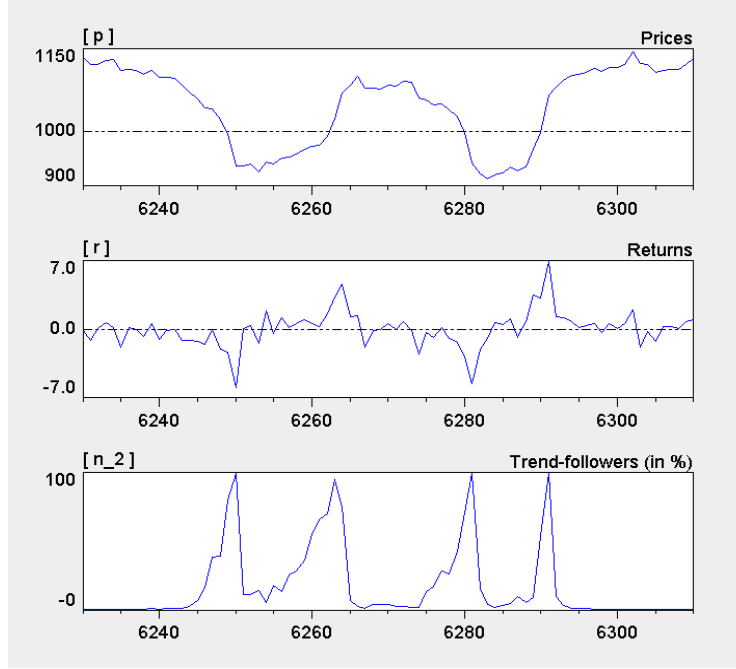


Figure 2: Extract from Figure 1.

The presentation of the model and its results by GH suggests that the changing market fractions in the bottom panel of Figure 2 are mainly caused by the differential fitness of trend-followers and EMH believers. This, however, is not quite true. As a matter of fact, over the whole time interval shown in Figure 2 the original discrete choice probability $\tilde{n}_{2,t}$ in (5) is equal to unity! The variations of the actual market fraction $n_{2,t}$ of the trend-followers have therefore to be exclusively ascribed to the dampening coefficient $\exp[-(p_{t-1} - p^*)^2/\alpha]$ in (6).

Responsible for this effect is the low(!) value of the coefficient $\alpha = 1800$. Rewriting the argument of the exponential function in the numerator as $-[(p^*)^2/\alpha] [(p_{t-1} - p^*)/p^*]^2$, it is easily checked that a dampening coefficient of 0.10 obtains when the price deviates by 6.4% from the fundamental value.⁷ A ten percent deviation of the price from p^* even reduces the dampening coefficient to 0.0038. Since the trend-followers drive the price in one direction, it follows from their strong risk aversion to already a moderate misalignment that this strategy can survive for only very few days. Concretely, only during 11 of the 81 days in Figure 2 are there more trend-followers than EMH believers on the market.

While Figure 2 is a qualitatively representative illustration of the phenomenon of volatility clustering, it is rather special in that the market price crosses the $p = p^*$ line

⁷ Computing $(p^*)^2/\alpha = 1000^2/1800 = 555.5$ and putting $x_{t-1} := (p_{t-1} - p^*)/p^*$, one has to solve the equation $0.10 = \exp(-555.5 \cdot x_{t-1}^2)$ for x_{t-1} .

no less than four times within the 81 days. Figure 1 demonstrates that usually the price stays away from the fundamental value over much longer spells of time, where the trend-followers have almost completely disappeared from the market. In fact, extending the simulations over 50,000 days we find that about 85% of this time the EMH believers form a majority of 99% and more; and they are in a minority only over less than 2% of this time. The exact numbers are reported in the first row of Table 2.

	cases of $n_{2,t}$ exceeding		
	1 %	50 %	90 %
APS+	15.33	1.80	0.59
APS−	15.56	1.91	0.59
NPS+	15.76	1.90	0.57
NPS−	14.06	1.66	0.52

Table 2: Quantiles of the shares of trend-followers under alternative shock scenarios (based on 50,000 observations).

Note: APS and NPS are additive and normalized price shocks, respectively. The random number sequence ‘−’ is obtained from the ‘+’ sequence through sign reversal.

In addition to the qualitative volatility clustering, GH are interested in the statistical properties of the return series of their model and how they compare to those of real data. To this end they concentrate on the autocorrelation functions (ACF) of the daily raw and absolute returns.⁸ As it should be for the raw returns, they exhibit no significant autocorrelation at longer lags. On the other hand, it will be evident from the discussion of Figure 2 that the autocorrelation coefficients are significantly positive at the first two lags, although the strong effects observed in this diagram are attenuated by the quasi random walk behaviour of the price that prevails most of the other time. The figures resulting from our 50,000 period sample run are documented in the first row of Table 3.

The statistically most attractive feature of the GH model is connected with the absolute returns: they are significantly positive not only for the short lags but for all long lags, too (likewise, see the first row in Table 3). On this basis, GH (p. 281) conclude that “although our model is only six dimensional it is able to generate apparent long memory effects”.

2.3. *A critical discussion of the long memory effects*

The quotation at the end of the previous subsection emphasizes the low dimensionality of the model. Nevertheless, it also suggests that actually all six dimensions contribute to

⁸ We omit the squared returns since their ACF provides no further insights *vis-à-vis* the absolute returns.

	sd	ACF raw returns at lag			ACF absolute returns at lag				
		1	2	100	1	2	5	50	100
original r_t	1.241	0.142	0.078	-0.008	0.210	0.117	0.063	0.054	0.055
$\tilde{r}_t \sim N(0, 1.241^2)$									
if $n_{2,t} > 0.90$	1.193	0.065	0.050	-0.004	0.131	0.114	0.075	0.062	0.062
if $n_{2,t} > 0.50$	1.148	0.020	0.007	-0.001	0.095	0.092	0.087	0.075	0.073
if $n_{2,t} > 0.01$	1.167	-0.002	-0.003	0.000	0.081	0.083	0.084	0.066	0.067

Table 3: Statistics of the original (r_t) and modified (\tilde{r}_t) return series.

Note: Based on simulation (APS+) over 50,000 periods, so that the Bartlett standard deviation of the coefficients is $1/\sqrt{50,000} = 0.0045$. ‘sd’ is the standard deviation of the returns. In the modified series \tilde{r}_t , the original r_t is replaced with random draws from the normal distribution $N(0, 1.241^2)$ if, and only if, at that date $n_{2,t}$ exceeds the indicated threshold.

the long memory effects. If one realizes that most of the time the vast majority of traders are EMH believers and the price dynamics is close to the one-dimensional quasi random walk (9), this statement is no longer so obvious. To check this issue we conduct a little experiment that reduces the role of the agents’ interactions in the model.

We take the return series r_t from the 50,000 period simulation and replace this value with another return \tilde{r}_t at all days where the fraction of trend-followers exceeds a certain threshold \bar{n}_2 of 90, 50 and 1 percent, respectively. This new return is drawn from a normal distribution whose standard deviation 1.241 is taken over from the original series. In this way the phases of the dynamic process when the price is near the fundamental value are filtered out, but only these.

The outcome of this replacement procedure is presented in the lower part of Table 3. It is clear that as the threshold \bar{n}_2 decreases and so excludes an increasing number of days that have generated the volatility clustering, the standard deviation of the artificial series \tilde{r}_t diminishes as well as the short-lag coefficients of $\text{ACF}(\tilde{r}_t)$ and $\text{ACF}(|\tilde{r}_t|)$. At all longer lags, however, the coefficients of the absolute returns do not follow this rule; if anything they are even higher than those for $|r_t|$. Table 3 thus indicates that it is essentially the EMH believers that produce the long memory phenomenon, and that not all of the model’s six dimensions are required for that.

Before proceeding with this discussion, let us consider the concrete numbers of the ACF coefficients that we have obtained in the first row of Table 3. After all, some of them are considerably lower than those reported by GH (2007, Table 2, p. 282). At the lags 1, 2 and 5 the coefficients of GH for the absolute returns are 0.193, 0.156 and 0.124, respectively; and from their correlogram one infers that at lag 50, $\text{ACF}(|r_t|)$ is greater than 0.10 (*ibid.*, p. 270).

To understand this variability in the statistical numbers it is useful to refer to eq. (9)

and rewrite the returns in a regime of EMH believers (when $n_{2,t} = 0$) as

$$0.01 \cdot r_t = \frac{r}{1+r} \frac{p^* - p_{t-1}}{p_{t-1}} + \frac{\sigma_\varepsilon \varepsilon_t}{p_{t-1}} \quad (10)$$

This expression shows that positive and negative deviations of the market price from the fundamental value $p^* = 1000$ make a difference. With a positive deviation of $p_{t-1} - p^* = 200$, say, an ε_t shock of one standard deviation $\sigma_\varepsilon = 10$ yields a return $r_t \approx 100 \cdot 10 / 1200 = 0.83\%$, whereas with a negative deviation of -200 the same shock raises the return to $r_t \approx 100 \cdot 10 / 800 = 1.25\%$. With deviations ± 400 , which according to Figure 1 are not too extraordinary after the first 3,500 days, the discrepancy is even more striking: 0.71% *versus* 1.67% .

In order to check whether the scaling issue in (10) has a bearing on the order of magnitude of the autocorrelation coefficients, we take the shock sequence $\{\varepsilon_t\}$ that constituted our stochastic sample run. Now, however, instead of adding the ε_t to the undisturbed \hat{p}_t in (APS), we subtract them from \hat{p}_t . Correspondingly, the first shock scenario will be referred to as (APS+) and this second one as (APS−).

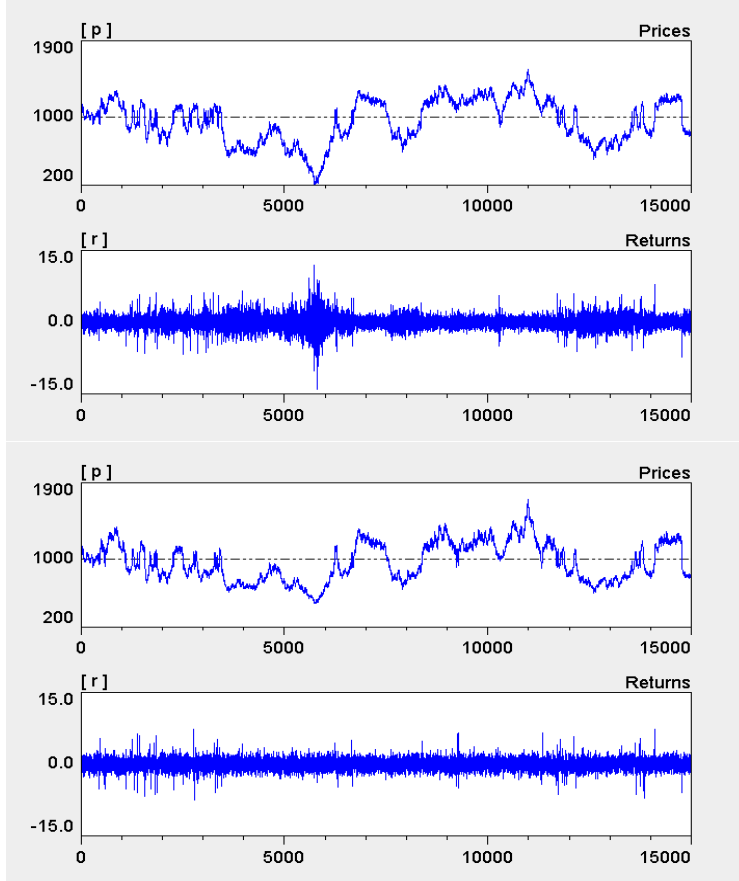


Figure 3: Simulations with additive price shocks (APS−) (upper two panels) and normalized price shocks (NPS−) (lower two panels).

The prices and returns of the first 15,000 days under (APS−) are plotted in the upper part of Figure 3. The pattern of the price series is an almost perfect reverse mirror image of the (APS+) simulation in Figure 1. This phenomenon goes along with very similar frequencies at which the trend-following strategy appears on the market; cf. the second row in Table 2.

The fact that around $t = 5,800$ the strong deviations of about 800 are now negative rather than positive leads to another kind of volatility clustering in the returns. Since in that phase the market is in a regime of EMH believers, it is clear from the discussion of eq. (10) that this is a pure price scaling effect without further economic content. We will return to this issue shortly.

Turning to the ACF, the first two rows of Table 4 compare these statistics for the two shock scenarios (APS+) and (APS−) over the 50,000 observations of our sample period (sd and ACF for (APS+) are reproduced from the first row of Table 3; “EMH + TF” indicates the simulations of the original model where generally both EMH believers and trend-followers are present). Except for the standard deviation of the returns themselves, the (APS−) realizations of the additive price shocks lead to higher coefficients, although those of $\text{ACF}(|r_t|)$ do not yet reach the abovementioned order of magnitude in the simulation by GH. This order is, however, attained (and even exceeded) if we follow GH and base the computations on the first 10,000 days only; see the third row in the table and recall GH’s coefficients 0.193, 0.156 and 0.124 for the lags 1, 2 and 5. The numerical examples demonstrate that the model exhibits a considerable variability in the summary statistics. Of course, this is largely explained by the fact that roughly 85% of the time, when $n_{2,t} \leq 1\%$, the price dynamics does not differ much from a random walk.

			ACF(r_t) at lag			ACF($ r_t $) at lag			
	sd	$\hat{\alpha}_H$	1	2	100	1	2	5	100
EMH + TF, APS+	1.241	3.59	0.142	0.078	−0.008	0.210	0.117	0.063	0.055
EMH + TF, APS− days 1 – 10,000	1.181	3.24	0.160	0.089	−0.005	0.254	0.159	0.098	0.088
	1.383	3.29	0.091	0.038	0.001	0.244	0.190	0.159	0.168
EMH only, APS−	1.054	4.37	0.001	0.001	0.003	0.109	0.108	0.109	0.103
EMH + TF, NPS−	1.100	3.72	0.150	0.082	0.001	0.174	0.066	0.001	0.001
EMH only, NPS−	0.996	5.98	−0.003	0.001	0.001	−0.002	0.000	0.001	0.002

Table 4: Statistics of returns under alternative shock scenarios.

Note: Based on 50,000 observations each (except for the third row). ‘sd’ is the standard deviation of the returns, $\hat{\alpha}_H$ the Hill estimator with a 5 percent tail.

As an aside, Table 4 additionally reports another aspect of the stylized facts of daily returns, which are the fat tails. They are conveniently measured by the Hill estimator $\hat{\alpha}_H$. As for real stock market data it typically ranges between 3 and 4, the table shows

that the model's volatility clustering implies a good match of this criterion, too.⁹

To identify the main cause for the long memory, we can now pick up the discussion of Table 3. Its conclusion that the phenomenon is essentially generated by the EMH believers can be most directly checked by putting $n_{2,t} = 0$ for all t , which amounts to simulating the quasi price random walk (9). Certainly, the autocorrelations of the raw returns must be insignificant then, and also the low-lag autocorrelations of the absolute returns diminish. The fourth row of Table 4, however, documents that at all longer lags the levels of $\text{ACF}(|r_t|)$ are maintained. Combining this result with the previous discussion, we have safely established that the long memory is indeed exclusively due to the EMH believers; that is, the other structural components of the model with the trend-followers and the evolutionary process do not contribute to it at all (except that the process marginalizes the role of the trend-followers).

Let us then go one step further and reconsider the nature of the price shocks. The additive shocks would be quite innocent if the prices remained within a relatively narrow corridor around the fundamental value. In the light of the wide variations of the market price that the model typically produces this specification is, however, no longer appropriate. To guard against the pure scaling effects that we have revealed, the price shocks should rather be normalized. Thus, we now replace equation (APS) with

$$p_t = \hat{p}_t (1 + \tilde{\sigma}_\varepsilon \varepsilon_t), \quad \varepsilon_t \sim N(0, 1), \quad \tilde{\sigma}_\varepsilon = \sigma_\varepsilon / p^* = 0.01 \quad (\text{NPS})$$

where (NPS) stands for normalized price shocks. Of course, the random number sequence $\{\varepsilon_t\}$ will be the same as in the simulations above, so that, in obvious notation, we have two additional shock scenarios (NPS+) and (NPS-).

The lower part of Figure 3 plots the price and return series resulting from the simulation with the shocks (NPS-). The differences from (APS-) in the upper part of the figure are plain to see. First, while the pattern of the price fluctuations is fairly similar, the negative deviations (especially) are much more limited, which is exactly what one will have expected. Second, the fluctuations of the returns are more limited, too. It might even be feared that the volatility clustering phenomenon has disappeared.

To check the latter, we first refer to Table 2 and note that the significance of the trend-followers has not much changed. The lower percentages, under (NPS-) *versus* (APS-), of the cases where $n_{2,t}$ exceeds the given thresholds is probably specific to the present random number sequence, since for (NPS+) the percentages tend to be somewhat higher than for (APS+). In any case, as a comparison of the second and fifth row in Table 4 shows, the first- and second-order autocorrelations of the raw returns under (NPS-) are

⁹ The computation of the Hill estimator presumes that the absolute returns $v_t := |r_t|$ are already rearranged in ascending order, $v_{i-1} \leq v_i$ for $1 \leq i \leq k := 50,000$. Specifying the tail of this series by the last m elements, it is then defined as $\hat{\alpha}_H = 1/\hat{\gamma}_H$ with $\hat{\gamma}_H = (1/m) \sum_{i=0}^{m-1} [\ln v_{k-i} - \ln v_{k-m}]$. Obviously, lower values of $\hat{\alpha}_H$ indicate a fatter tail of the data.

only slightly lower than under (APS−). Likewise, the Hill estimator $\hat{\alpha}_H = 3.72$ is still indicative of a fat tail.

The major difference between (NPS−) and (APS−) are the autocorrelations of the absolute returns. They are considerably lower at the first and second lag, though they continue to be significantly positive. Most importantly, however, the insignificant coefficients at the higher lags ascertain that the long memory phenomenon has faded away. To round out the argument, the last row in Table 4 shows the same insignificance for a perfect market regime of EMH believers.

In a short summing up it can be said that while the GH model is able to produce volatility clustering in returns, its long memory effects in the absolute returns are quite artificial since they rest on additive price shocks that reasonably need to be, but are not, normalized. On the other hand, after normalizing the price shocks it will not be easy to design another mechanism that can recover the long memory. One may infer this from the unsatisfactory estimation results by Amilon (2008) in the present class of models with, as he emphasizes (p. 359), multiplicative noise in prices.¹⁰

3. The He–Li model

3.1. *Formulation of the model*

The model by He and Li (2007; HL henceforth) that is to be discussed in this section appears to be fairly different from the GH model at first sight. Recall that in the GH model the demand of fundamentalists and trend-followers is proportional to their expected excess profits, and that the weights of the two components of demand are varying over time as the market fractions of the two groups undergo an evolutionary process. In the HL model, by contrast, the market fractions are fixed. However, the variability that is lost in this way is to some extent reintroduced by the assumption that in the formulation of demand the conditional variance of the trend-followers is endogenously changing, while that of the fundamentalists remains constant.

The demand mechanisms in the two models are therefore not that different. Also the different determination of the market price, which in HL is adjusted in the direction of excess demand as opposed to the market clearing in GH, should be of limited significance—at least once the models are calibrated such that they generate cyclical trajectories in their deterministic skeleton, which both the GH and HL model actually do.

To begin with the speculative demand $z_{h,t}$ in the HL model, it has just been mentioned that GH’s assumption of a uniform and constant variance of the excess returns is dropped.

¹⁰ Actually, Amilon’s modelling approach is even more elaborated than the present setting since he also includes a contrarian trading strategy and time-varying conditional variances in (1).

Hence σ^2 in (1) is replaced with a variable $\sigma_{h,t}^2$ and the equation becomes,¹¹

$$z_{h,t} = \frac{E_{h,t}[p_{t+1} + y_{t+1} - (1+r)p_t]}{a \sigma_{h,t}^2} \quad (11)$$

The fundamentalists adopt the same forecasting rule as in GH, except that instead of p_{t-1} they use the predetermined price p_t as the most recent price information. In addition, HL assume a variable fundamental value, $p^* = p_t^*$ (see below). For better comparison with (3), the price expectations of the fundamentalists may be written as,

$$E_{1,t}(p_{t+1}) = p_t^* + (1-\alpha)(p_t - p_t^*) \quad 0 \leq \alpha \leq 1 \quad (12)$$

Obviously, $(1-\alpha)$ corresponds to the coefficient v in (3).

The expectations of trend-followers are a bit more ambitious than in GH. Here these traders are supposed to extrapolate the distance of the observed price from a long-run moving average u_t , where the latter is computed over an infinite horizon with a geometric decay parameter δ . The variance v_t of prices is estimated in a similar manner, with the same coefficient δ for the geometric decay process. Taken together we have,

$$\begin{aligned} E_{2,t}(p_{t+1}) &= p_t + \gamma(p_t - u_t) \\ u_t &= \delta u_{t-1} + (1-\delta)p_t \\ v_t &= \delta v_{t-1} + \delta(1-\delta)(p_t - u_{t-1})^2, \quad 0 \leq \delta \leq 1 \end{aligned} \quad (13)$$

Turning to the conditional variance $s_{h,t}^2$ of the excess returns in the denominator of (11), let $r^a = 250 \cdot r$ be the annual rate of interest (with respect to 250 trading days), σ_1^2 the unconditional variance of the price, and $b > 0$ another coefficient. Then HL assume that the two groups of agents perceive their variance $s_{h,t}^2$ as follows:¹²

$$\begin{aligned} \sigma_{1,t}^2 &= (1+q)\sigma_1^2, \quad q = (r^a)^2 = (250 \cdot r)^2 \\ \sigma_{2,t}^2 &= (1+q+bv_t)\sigma_1^2 \end{aligned} \quad (14)$$

To complete the specification of excess demand, expectations of the dividend payments y_{t+1} are $r\bar{p}$, where \bar{p} is the expected long-run fundamental value. The market fractions of fundamentalists and trend-followers, n_1 and n_2 , have already been announced to be fixed. On the whole, total excess demand $z_t = z_{1,t} + z_{2,t}$ derives from (11) as,

$$z_t = \frac{1+m}{2} \frac{E_{1,t}(p_{t+1}) + r\bar{p} - (1+r)p_t}{a(1+q)\sigma_1^2} + \frac{1-m}{2} \frac{E_{2,t}(p_{t+1}) + r\bar{p} - (1+r)p_t}{a(1+q+bv_t)\sigma_1^2} \quad (15)$$

where m is defined as $m := n_1 - n_2$. Shifts in the weights of the two demand components are here brought about by the time-varying term bv_t . In principle, the (modulus of the)

¹¹ The notation in HL also allows the risk coefficients to differ across the agents. We disregard this option since in their calibration HL do not exercise it, either.

¹² Regarding the determination of the coefficient q in HL's (2007) equations (2.9) and (2.10), see their footnote 12 on p. 3403. Notice in this respect that their symbol r is the annual interest rate and so corresponds to r^a in (14), while they write $(R-1)$ for our symbol r of the daily interest rate.

demand of trend-followers can be arbitrarily small relative to fundamentalists, but not the other way around. A common feature with the GH model is that the influence of the trend-followers increases in times of tranquillity or steady price movements, though in HL the mechanism is more direct than in GH, where it involves the differential profits in the evolutionary process.

Generally, the excess demand need not be zero in (15), that is, the market is allowed to be in disequilibrium. The agents can nevertheless realize their plans with the aid of a market maker, who serves as a buffer. His role is to take a long position (when $z_t < 0$) to absorb the shares that are in excess supply, and to take a short position (when $z_t > 0$) to provide the market with the shares that are in excess demand. At the end of period t , after all these transactions have been carried out, the market maker adjusts the price for the next period in the direction of the observed (positive or negative) excess demand z_t with a speed $\mu > 0$. Random factors in demand, the transaction process or the price quotes themselves are treated in exactly the same way as in GH, namely, by introducing i.i.d. additive price shocks (APS). So the price in period $t+1$ is determined as,

$$\hat{p}_{t+1} := p_t + \mu z_t \quad (16)$$

$$p_{t+1} = \hat{p}_{t+1} + \sigma_\varepsilon \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \quad (\text{APS})$$

The model is closed by the exogenous stochastic law that governs the evolution of the fundamental value, for which HL employ the standard assumption of a random walk. Remarkably, however, it is here stated in a normalized form,

$$p_{t+1}^* = (1 + \sigma_\phi \phi_t) p_t^*, \quad \phi_t \sim N(0, 1) \quad (17)$$

(The event $\sigma_\phi \phi_t < -1$ is so unlikely that no extra qualification is made for it.) Even without any suspicion of the possibly problematic consequences of the additively specified price shocks, the unequal treatment of the shocks in (APS) and (17), which is left uncommented by the authors, appears a bit peculiar. One would rather expect that the shocks are both either additive or normalized.

The deterministic skeleton of the model is a three-dimensional difference equations system, which by virtue of its highly nonlinear nature can generate rich dynamic phenomena. For their investigation of the stochastic dynamics, HL are exclusively concerned with the set of parameters collected in Table 5. Their discussion makes it very clear that the adjustment period is one day. Thus, the daily interest rate r corresponds to an annual rate of 5%, and the values of the standard deviations σ_ϕ and σ_1 derive from an annual volatility of the fundamental value of 20%.¹³

¹³ Table 5 combines HL's Table 1 and the additional information in their footnotes 12 and 13 (HL, 2007, pp. 3403f). In particular (in the present notation), $r = 0.05/250 = 0.0002$, $\sigma_\phi = 0.20/\sqrt{250} = 0.01265$, $\sigma_1^2 = (0.20 \cdot \bar{p})^2/250 = 1.60$.

a	α	γ	δ	m	\bar{p}	r	σ_1^2	b	μ	$\sigma_\varepsilon/\bar{p}$	σ_ϕ
0.80	0.10	0.30	0.85	0.00	100	0.0002	1.60	1.00	2.00	0.010	0.01265

Table 5: Numerical parameters of the HL model.

3.2. *The role of the price shocks*

The top panel in Figure 4 shows the time path of the market price in the HL model over the first 15,000 days. The sequence of the shocks to the fundamental value in (17) is, by the way, the same as for the price shocks in the simulation of the GH model. Indeed, the broad pattern of the motions of p_t after the first 5,000 days is qualitatively similar to the top panel in Figure 1. Given that in the HL model p_t does not persistently disconnect from p_t^* (as illustrated in the bottom panel of Figure 4 and, over a longer period, in HL, 2007, Figure 4 on p. 3407), the wide fluctuations of the price are here basically caused by the exogenous random walk of the fundamental value. Note that also in the GH model the price dynamics is, over long passages of time, reduced to a (quasi) random walk, although this is an endogenous mechanism since the random walk is put into operation when the EMH believers happen to form the overwhelming majority on the market. Anyhow, the occasional reappearance of the trend-followers in GH causes their price fluctuations to be more limited than in the HL model.

The large deviations of p_t from the long-run fundamental value $\bar{p}=100$ at around $t=5,800$ point to a first problem with the additive price shocks (APS). Another sequence of the shocks ϕ_t to the fundamental value may drive the price to very low values. While by the normalization design in (17) the fundamental value itself will always stay positive, there are good chances for the price p_t to turn negative. For the simulations shown in Figure 4 this happens around $t=26,600$, and experiments with other shock sequences lead to a similar result.¹⁴ Hence for the model to be well-defined, the assumption (APS) would have to be complemented by a rule that takes the negativity problem into account.

As in the sample run studied by HL (2007; see the left bottom panel in their Figure 2, p. 3405), the return series in the second panel in Figure 4 shows clear evidence of volatility clustering. This is statistically confirmed by its autocorrelation patterns. The coefficients of the raw returns are (nearly) all insignificant and need not be further discussed here. The ACF of the absolute returns is plotted as the upper bold (blue) line in Figure 5 (it corresponds to the bottom panel in the middle of Figure 3 in HL, 2007, p. 3405). There can be no doubt about the significance of these coefficients and the long range dependence they indicate.¹⁵

¹⁴ HL do not encounter this problem since, together with an initial transition period, they only report results over the first 6,000 days.

¹⁵ For completeness it may be remarked that there is a decay in the coefficients, though it is

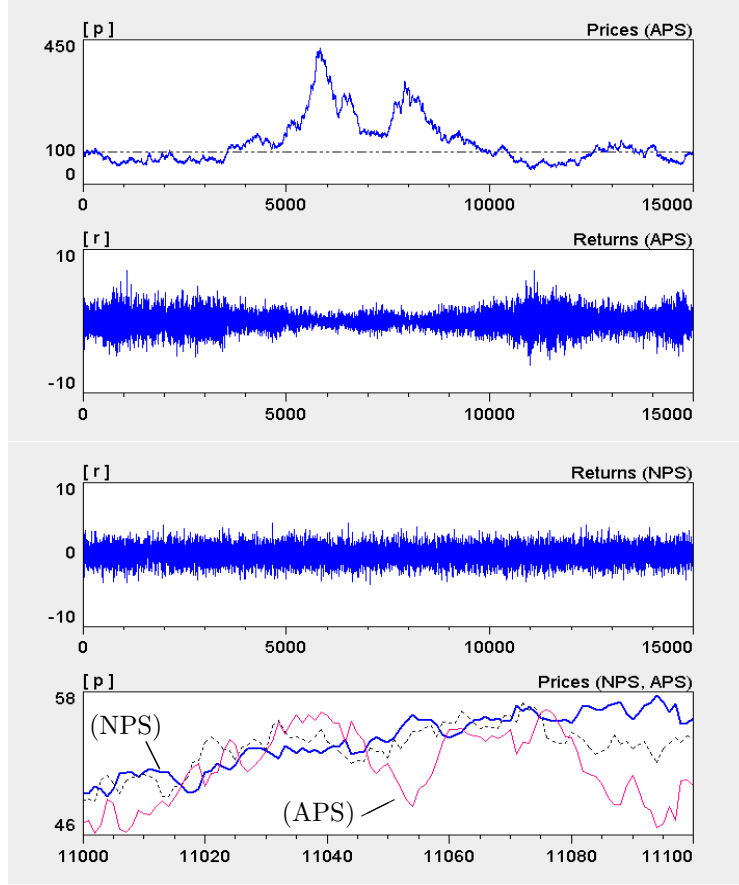


Figure 4: Simulations of the HL model under additive and normalized price shocks.

Note: Bold solid line (blue) in the bottom panel originates with (NPS), thin solid line (red) with (APS); the dotted line depicts the fundamental value.

The line designated APS* in Figure 5 illustrates that $\text{ACF}(|r_t|)$ is sensitive to the specific realization of the shocks to the fundamental value, in the sense that not the qualitative pattern, but the general level, of the autocorrelations is dependent on whether the deviations of p_t from $\bar{p} = 100$ are positive or negative. Actually, the coefficients of the APS* line are computed for the subperiod $[3500, 9700]$, over which $p_t > \bar{p}$. Partly p_t is here two or even four times higher than \bar{p} , so that the impact of the additive price shocks is two or four times weaker. This means that the relative price changes and thus the returns tend to be much smaller over this period, as it is clearly seen in the second panel of Figure 4. A further implication, then, is that also the coefficients of $\text{ACF}(|r_t|)$ over this period lie below those of the entire sample period.

extremely slow. One needs at least 200 lags to recognize that $\text{ACF}(|r_t|)$ decreases in a, practically, linear way, and even at lag 1,000 the coefficient is still in the region of 0.10 (these computations are based on 25,000 observations).

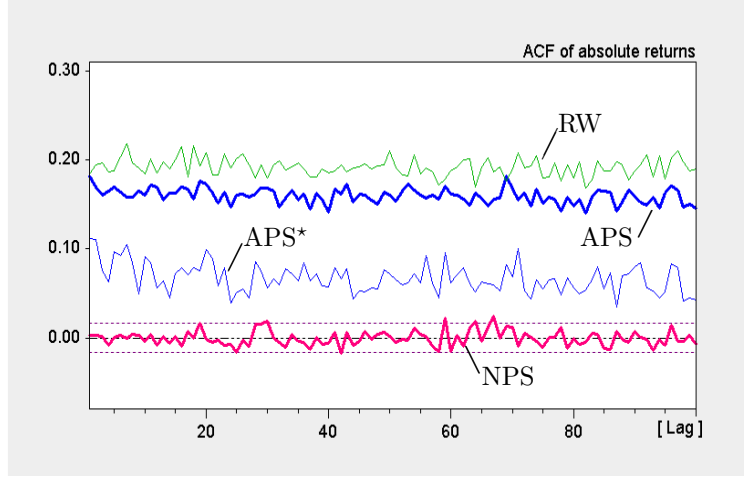


Figure 5: ACF of absolute returns in the HL model under different price shock scenarios.

Note: On the basis of 15,000 observations, except for APS*, which is based on the subinterval [3500, 9700]. The dashed lines are ± 2 times the Bartlett standard deviation of the coefficients. ‘RW’ results from a price random walk with standard deviation 1.15.

HL (2007, p. 3406) contend that three factors work together to produce the volatility clustering and the apparent long memory: (1) the nature of the endogenous variations of the conditional variance $\sigma_{2,t}^2$ of excess returns on the part of the trend-followers; (2) the noisy fundamental process; (3) the noise in the determination of the market price. In fact, their arguments appear so persuasive that the three points almost read as a recipe for generating the stylized facts in other agent-based models.

After the discussion of the GH model, however, the litmus test is now the normalization of the price shocks. Correspondingly, we again respecify (APS) as,

$$p_{t+1} = \hat{p}_{t+1} (1 + \tilde{\sigma}_\varepsilon \varepsilon_t), \quad \varepsilon_t \sim N(0, 1), \quad \tilde{\sigma}_\varepsilon = \sigma_\varepsilon / \bar{p} = 0.010 \quad (\text{NPS})$$

Certainly, our simulations with (NPS) adopt the same shock sequences $\{\varepsilon_t\}$ and $\{\phi_t\}$ as before.

The bottom panel of Figure 4 illustrates an immediate consequence of the modification (NPS). Together with the fundamental value p_t^* (the dotted line), the panel depicts the prices resulting from (APS) and (NPS), respectively, over a period where all prices are considerably below \bar{p} . In this stage of the process, the price shocks in the (APS) scenario have a stronger impact than under (NPS). This in turn triggers stronger reactions of the trend-followers, so that their destabilizing potential leads to larger deviations of the price from the fundamental value and also to larger price changes, or wider fluctuations of returns. On the other hand, over periods where the prices move persistently above \bar{p} , the picture will be reversed. Hence, so far, the normalization of the price shocks (NPS)

indicates no straightforward effects on the returns.

Let us therefore consider the returns resulting from (NPS) in the HL model directly, which is done in the third panel in Figure 4. As in the bottom panel in Figure 3 for the GH model, there are clearly no more signs of volatility clustering. This statement is authenticated by the ACF of the absolute returns, which is depicted as the lower bold (red) line in Figure 5. As the confidence band of the dotted lines shows, virtually all coefficients have become insignificant.

For the GH model it was revealed that its stylized facts were due to the long periods of time over which the model nearly behaves like a random walk (with purely additive increments, that is). By contrast, in the HL model a similar feature is not that easy to detect. We thus try more indirect evidence, in that we simulate a pure random walk of the price, $p_{t+1} = p_t + \sigma_{RW} \phi_t$ with $\phi_t \sim N(0, 1)$, compute its ACF of the absolute returns, and compare it to the ACF of the original HL model. Since the ACF of the random walk shifts upward as the standard deviation σ_{RW} is increased, we scale the latter such that $\text{ACF}(|r_t|)$ at the first lag coincides with that of the HL model (which is achieved by $\sigma_{RW} = 1.15$).

The resulting function is the topmost thin solid (green) line in Figure 5. With its extremely slow decay, it looks very much like the $\text{ACF}(|r_t|)$ of the HL model under (APS). A slightly lower σ_{RW} would actually shift it downward, so that the two ACF could no longer be significantly distinguished from each other.

The ACF structure of the original HL model can now be reconsidered as follows. (1) The random walk in the fundamental value ensures a sufficient overall variability of p_t , which chases p_t^* as a moving target. (2) In the absence of price shocks, the unit root behaviour of the fundamental value would essentially carry over to the market price and even to the returns; see the low-lag $\text{ACF}(r_t)$ and $\text{ACF}(|r_t|)$ near unity in the top row of Figure 3 in HL (2007, p. 3405). (3) The random shocks to the price can then be viewed as a short-term noise that, in particular, renders the ACF of the raw returns insignificant (as pointed out by HL, p. 3406). At the same time, this noise also reduces the low-lag ACF of the absolute returns and raises the ACF at the long lags. However, as we have seen, for this phenomenon to come about it is absolutely necessary that the price shocks are additive. If they are normalized in the same way as the shocks to the fundamental value, as for lack of any other evidence it would be conceptually appropriate, the pleasant features of long memory and volatility clustering cease to exist.

4. Conclusion

The paper has reassessed two of the most promising small-scale agent-based models from the literature that, featuring fundamentalists and trend-followers, purport to be capable of matching the ‘stylized facts’ of volatility clustering and long-range dependence in the (daily) returns of a risky asset. It revealed that the corresponding autocorrelation

(AC) pattern of the absolute returns is basically due to the models' particular specification of the random shocks to the market price, which are supposed to be additive. Our criticism is that this assumption loses its innocence and is even artificial if, as it is the case in both models, the price happens to undergo wide fluctuations.

The problem is best understood if we consider a random walk of the price (with additive innovations). As the returns r_t are then given by dividing these independent shocks through the current level of the price, the absolute values of r_t will over longer passages of time be persistently above or below their mean. This implies significantly positive ACs of $|r_t|$, which are only slowly declining as the lag length increases. By construction, decisive for these effects are the (slow) variations in the denominator of the definition of the returns.

With their additive price shocks, the same mechanism is also present in the two models here discussed and thus at least partially responsible for their observed long memory. Now, however, the denominator mechanism is combined with the variability in the numerator, which (mainly) reflects the models' internal structure. So the dynamic feedbacks in the numerator may make a contribution as well, and this is indeed what the descriptions and evaluations of the models suggest. An easy way to check the contribution of the structural part of the models is to eliminate their denominator mechanism. That is, the additive price shocks are replaced with innovations that are proportionally scaled to the current level of the price. After all, in the absence of any other evidence and knowing of the possibly large price fluctuations, this normalization is the obvious and appropriate way to model price perturbations. The main result of the paper was that this elementary respecification causes all the long memory effects in the ACs of the absolute returns to vanish.

For these ACs it may thus be briefly said that, in the definition of the returns, the denominator mechanism, which originated with an artificial specification of the models' stochastic noise, has clearly dominated the numerator mechanism, which reflects the models' internal structure.¹⁶ Despite its broad scope for price variability, the numerator mechanism did not prove sufficient to generate the desired positive ACs of $|r_t|$ with their slow decay. Explaining them primarily by the endogenous interactions between the groups of, in some sense, fundamentalists and trend-followers still remains a challenging task.

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¹⁶ I owe this distinction between the numerator and denominator mechanism and the observation of the latter's dominance to Cars Hommes.

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